A hands-on approach to studying quadratic functions emphasizes the engineering design process.

JUMPI
TO QUADR

BONNIN STUDIO/STOCKSY

NG
ATIC MODELS

Devon Gunter

It is no easy feat to engage young people with abstract material as well as push them to greater depths of understanding. Add in the extra pressures of curriculum expectations and standards and the problem is exacerbated. Projects designed around standards and having multiple entry points clearly offer students the best opportunity to engage with otherwise daunting material and construct rich understanding (Wheatley 1991).

A project I have used in my classes addresses these issues. In this project, students explore how high a person can jump. They work in groups to design experiments and collect jump data with which they develop assumptions related to projectile motion. The project encourages students to explore topics in algebra, although suggestions for extensions in a calculus course are offered below. Students actively engage in the study of quadratic functions through this hands-on approach that emphasizes the engineering design process (see Jameson n.d.). Using this design process in my classroom has promoted curiosity, improved self-confidence, and encouraged resilience in my students.

START WITH YOUR FEET ON THE GROUND

In general, the vertical distance that a person can jump is directly related to the force that she can generate with her legs and inversely related to her mass. To mitigate any risk of disengaging students because of concerns over weight issues, the formal development of these topics can be safely omitted, but avenues for meaningful exploration can be provided in a calculus course. The position of a person's center of mass, relative to time, forms a parabolic curve during the course of a jump. Students may choose to explore the effects of varying methods of jumping (e.g., vertical or horizontal, bent knees or straight legged, stationary or running start) as well as varying methods of data collection, both analog and digital.

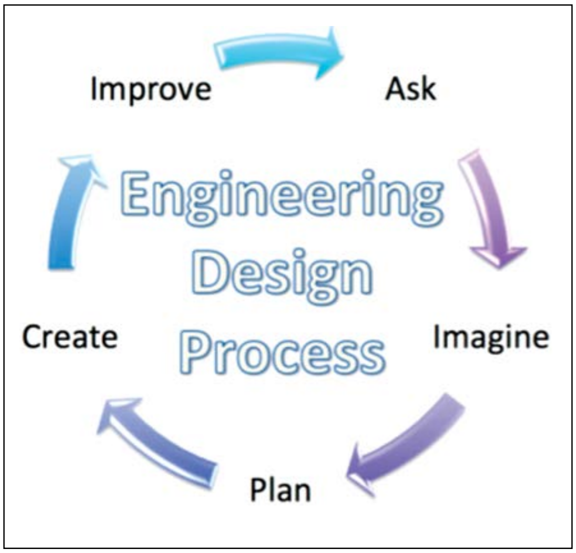

Fig. 1 The engineering design process is iterative.

THEN JUMP

This project is divided into several parts, each corresponding to a step in the engineering design process (**fig. 1**): define the problem (ask), brainstorm ideas (imagine), plan a solution (plan), make a model (create), and test the model and redesign as necessary (improve). In my class, I incorporate modeling into many topics, and our exploration of quadratic models in this project is mediated by this fact. A lesson such as this one can be used as an introduction to modeling, but some adaptation to meet the needs of individual classrooms is likely necessary.

Define the Problem (Ask)

To motivate our discussion, I ask this question: "How much hang time can a professional basketball player get?" ("Hang time" is the amount of time a person is airborne.) Asking this question often leads to a class discussion about who was (or is) the greatest basketball player—which, as a general rule, is a sign that the class is engaged with the problem. Showing a video of Michael Jordan's first dunk in round two of the 1987 NBA Slam Dunk Contest, wherein he jumps from the free throw line to dunk the basketball (available at https://youtube/ZUYQPTKwsT4?t=53s), further motivates meaningful conversation. Several comments and questions immediately arise:

- How did he do that?
- Does the run help him jump higher?
- There's no way that's real!
- LeBron is still better . . .

The comments and questions generated in this conversation will propel student explorations. It is important to help the students develop the notion that the hang time individuals can get has nothing to do with how far they jump but, rather, with how high they can jump. Students are often resistant to this fact, and who can blame them? They have anecdotal, empirical evidence that they can jump higher if they have a run-up than if they are stationary. One trick I have used to demonstrate the fact that hang time depends only on vertical height is to have a student with a stopwatch measure the amount of time it takes for an eraser to fall to the floor when I drop it from desk height and again when I push it off my desk. If enough class time is available, letting students discover this fact for themselves gives them a much more meaningful experience.

Brainstorm Ideas (Imagine)

Students work in groups of three to five to brainstorm ways to figure out how much hang time and height an individual can get in a jump. It is important to monitor students' progress but not offer specific suggestions or advice—the students must take ownership of their method. At this point, I usually have to redirect students to consider the problem; many still want to debate various topics in basketball. The goal of the "imagine" phase is for students to develop a list of key words and questions that will guide their approach.

During this phase, students also decide what tools they will need to answer the question. This list typically includes rulers and stopwatches, but occasionally students come up with more interesting or sophisticated tools such as chairs, iPads, and slow motion cameras (some of which I have and will permit them to use; most I do not). A few groups consider the processes that may be useful in addressing the question. In the spirit of collaboration, my groups designate one person as an ambassador, whose job is to share ideas with the other groups.

Devise an Experiment (Plan)

In my class, this phase is the longest of the project, and thus I set it as part of the daily homework. I encourage students to research answers to their questions and seek additional information that may help them. Each student group creates, types up, and submits for approval an experiment plan, which should include their relevant research findings and a short description of the design of their experiment, the kind of data they expect to glean, and possible ways to test the model. This process prepares students with at least a rough understanding of the kinds of data they will be generating.

I usually give myself some time (by spreading the activity over more than one class period) to obtain any of the more exotic measurement tools that have been requested—and that I believe may be useful. If this activity must be done in one class

period, providing a limited selection of tools at the start of the activity and omitting the requirement for a plan proposal will significantly shorten the time required to complete it. Not having the ambassadors share plans with the other groups may further reduce the time required, but at the expense of fewer iterations of the design process.

BONNIN STUDIO/STOCKSY

IN THE SPIRIT OF COLLABORATION, ONE PERSON IS AN AMBASSADOR, WHOSE JOB IS TO SHARE IDEAS WITH THE OTHER GROUPS.

Make a Model (Create)

The next class period, I provide resources, as I am able, to accommodate students' study; typically this includes tools to measure hang time directly (stopwatches, timers, etc.) and indirectly, by way of height (meter sticks, large format paper, etc.). I try to offer more choices than were asked for; some groups may have to change their data collection method fairly quickly, and having options available can spark creative solutions, such as using textbooks to approximate jump height. It may be appropriate to work with the groups to facilitate the collection of meaningful data within the constraints of their design proposal. Keep in mind that "meaningful" in this context may not mean "correct" or even closer to a "solution," but instead that the data collected makes sense (e.g., the students are not recording time data as length data).

Groups work through their data collection in numerous ways. Some try to approximate height with a meter stick while simultaneously timing with a stopwatch; others tape a large sheet of paper to the wall and have the jumper try to leave a mark at her highest point while being timed. Some groups return to the plan phase or adopt the approaches of their peers when their initial plan fails to produce the kinds of results they predicted. See the appendix, "A Classroom Scenario," for a narrative of a typical classroom experience. (A second scenario is available as a supplementary file at **www.nctm.org/mt**.)

Once the groups have collected their data, they use the information to attempt to create a model. At first, this model is only graphical or a rough illustration. Some groups decide to gather their data in multiple ways and work to reconcile any discrepancies. The goal is for students to flesh out their model using both symbolic and graphical representations and provide descriptive wording. In my class I permit use of calculators and computers, which can be used to help students identify unknown coefficient values through trial and error and to aid with graphing. Students also create an exposition that describes their data and poses any questions or strategies their data inspires. This exposition includes a table of the raw data as well as their model.

Some groups get stuck when their graphed points look random because of measuring inaccuracies. Some groups combine their data to try to find the pattern. A few groups, thinking that the relationship is linear, begin finding the slope of the line of best fit. Others, realizing that it is difficult to find a pattern in data that is closely grouped, begin recording data from smaller jumps. Wonderful insights such as "every term has to have a variable because the height is zero if we don't jump at all" and "linear equations don't bend" start filling the air, and the groups begin to feed off of one another's epiphanies.

The students test their models according to their plans. Nearly every group records an additional jump or two and tries to put that data into their model. Not all of the models are great ones or even good ones. Some models are completely wrong, but the groups have very little problem realizing whether their model is incorrect. One of the groups used only time data in creating its model

but immediately realized the error when the group members revisited their plan.

Test and Redesign (Improve)

The engineering design process is iterative. Therefore, students (ideally) would repeat these steps at least one additional time, implementing the insights from the previous iteration into both their designs and their experiments. This happened for all but one group, whose students were then sent off as peer-coaches to the other groups. Students identified deficiencies in their model and brainstormed causes and possible corrections. On occasion students needed to be redirected to the motivating question ("How much hang time can a professional basketball player get?") to determine that they were still working toward answering the driving question. Thus the cycle began anew. For a project such as this one, especially with the regular influx of peer coaches, the subsequent iterations came more quickly and were much more fruitful than the initial one. After two additional iterations every group had a good model for the data.

BONNIN STUDIO/STOCKSY

A BOOST FROM TECHNOLOGY

The limiting factor in student experiments is typically the precision of the measuring tools. Problems with the resolution of the tools can be overcome using technology. Myriad apps are available in both the iOS and Android markets for taking video of the jumps and for finding the times of initial lift-off and landing, the maximum height of the jump, or both.

In my class I have used the Vernier Video Physics™ app (http://www.vernier.com) with great success, although there are a number of other suitable apps in the Apple App Store. Using Vernier Video Physics, groups are able to record video of a student jumping. The app allows students to plot points on the video against time and generate graphs from those points. Students can also draw data from the app that is far more precise (especially hang time data, given that iOS devices record at rates of 30 frames per second or greater, depending on the model of the device). Calculator-based rangers (typically sonar devices) and photogates (infra-red event timers) can also be used to improve measurement results.

These devices may allow students to get fairly precise answers to the question of "how much hang time" but do little for the question of "how high." Therefore, they do not undercut the modeling process. These technology tools are not required for the project and only serve to provide more accurate measurements.

SLAM DUNK

Once all groups are satisfied with their models, we return to the guiding question: "How much hang time can a professional basketball player get?" Students discuss strategies that worked as well as those that were productive failures. To wrap up, I show a clip from the Fox Sports Network show *Sport Science,* episode 1, "Hang Time" (Fox Sports Network 2007).

This project can be extended into the realm of calculus in a number of ways. The most accessible is by finding velocity and acceleration functions

CONNECTING ALGEBRA TO REALITY WHILE SIMULTANEOUSLY PROVIDING LEARNING OPPORTUNITIES CAN SUPPORT MEANINGFUL DISCOURSE.

from their model (first and second derivative, respectively). One of my favorite modifications is to ask if we can experience weightlessness by jumping on Earth, then finding the force required for a 150-lb. person to achieve at least 0.1 seconds of weightlessness. The result of this project is that students actively engage with the application of mathematical concepts and standards by way of iterative improvement of self-developed models. Students are further pushed to document their progress formally and engage in meaningful intergroup and intragroup discourse. By doing these things, students are able to scaffold their learning by building on and extending their prior knowledge and to deal with multiple representations of the same mathematical concepts. This project focuses on the parabolic functions, measurement, modeling, and applications. Extensions provided include calculus topics (calculation and meaning of first and second derivatives, continuity, and applications) as well as integrating topics in physics (the relationship between force, velocity, mass, and acceleration).

Many students view math classes such as algebra as disconnected from the world around them. Connecting algebra to reality while simultaneously providing meaningful learning opportunities can be achieved through use of engaging activities, such as the one presented here, that support meaningful discourse. Possibly best of all, projects like these reengage students in math class by breaking the often passive nature of their learning experiences.

BIBLIOGRAPHY

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Fox Sports Network. 2007. "Hang Time." Episode 1 of *FSN Sport Science.* Uploaded Oct. 23. https://youtube/vZqVq5LrdQQ

Jameson, Skylur. n.d. "ABET Definition of Design." http://myweb.wit.edu/jamesons/Courses/MecDesign/lec3.pdf

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

———. 2014. *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Vernier Software and Technology. 2016. Video Physics app. http://www.vernier.com

Wheatley, Grayson H. 1991. "Constructivist Perspectives on Science and Mathematics Learning." *Science Education* 75 (1): 9–21. doi:10.1002/sce3730750103

APPENDIX: A CLASSROOM SCENARIO

Example 1

Three students are working together on the project. They decide that they will create a model by recording approximate jump height by having one group member (for this example Amy) read the jump height of another student (Brad) using a yardstick, while the third student (Christine) attempts to measure the duration of Brad's jump with a stopwatch. The group repeats this experiment several times to collect plenty of data. (See **table 1**.) The group members, not sure of where to begin, decide to plot their data. (See **fig. 2**.) The group notices that the relationship appears to be linear, but it is hard to tell, given how wildly the data seem to fluctuate. Brad takes it upon himself to try to place a best fit line, but he is unable to do so to the satisfaction of the rest of his group.

Table 1 Brad's Jump Data

t (seconds)	h (inches)
0.49	16
0.54	15
0.56	11
0.57	17
0.52	18
0.54	18
0.51	10
0.53	15
0.55	15
0.53	15

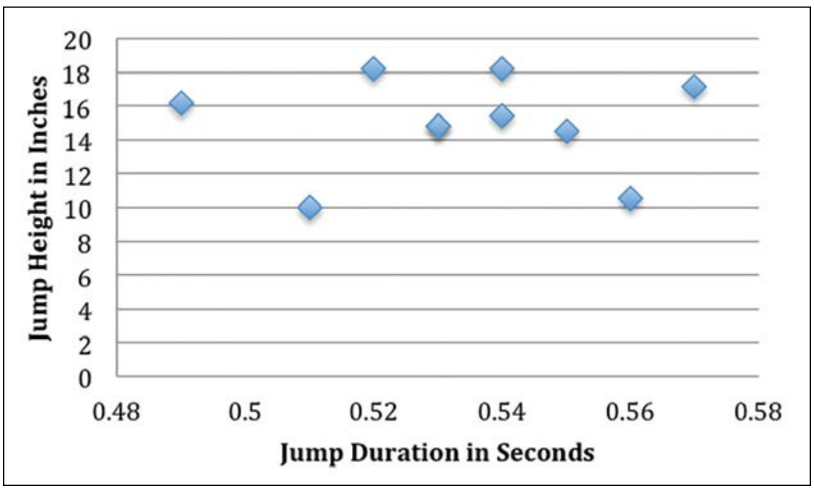

Fig. 2 Brad's original jump data look very weakly correlated..

Blaming their problems on the inaccuracy of their measurements, the group looks for ways to obtain more precise measurements. Instead of visually approximating Brad's jump height, the group fixes the yardstick higher on the wall and has Brad place a sticky note on it at the peak of his jump. The group also uses a calculator-based ranger to get a more accurate measure of Brad's jump duration. (See **table 2**.) Again the group members collect several data points and plot them. (See **fig. 3**.)

Although they are not convinced, the group members now suspect that the data may not be linear. Amy realizes that Brad's jumps are actually parabolic and the group changes its approach and tries $h(t) = k(t - 0)(t - c)$, where k is unknown and c is the duration of the jump. Christine conjectures that Brad's maximum height occurred at the midpoint of his jump. By plugging in their best data, the group members obtain the values for k shown in **table 2**.

Taking the average of these k-values, –190.8, the group obtains the model $h(t)= -190.8t(t - c)$. They test their model by having Brad jump a few more times and plug in c for the jump duration and $c/2$ for t to seeing if they can obtain the jump height, which they do (within a reasonable margin of error). See **table 3.**

Satisfied with their model, the group members compare their model to that of their peers, and the class as a whole discusses any discrepancies.

Table 2 Brad's Jump Values (Second Experiment)

t (seconds)	h (inches)	k
0.533	13.5	–190.081
0.489	11.4	–190.698
0.488	11.3	–189.801
0.487	11.2	–188.895
0.495	11.7	–191.001
0.484	11.1	–189.536
0.530	13.5	–192.239
0.520	12.8	–189.349
0.486	11.3	–191.366
0.484	11.4	–194.659

Table 3 Test of Model against Additional Observations

t (seconds observed)	h (inches observed)	h (inches predicted)
0.526	13.5	13.2
0.496	11.9	11.7

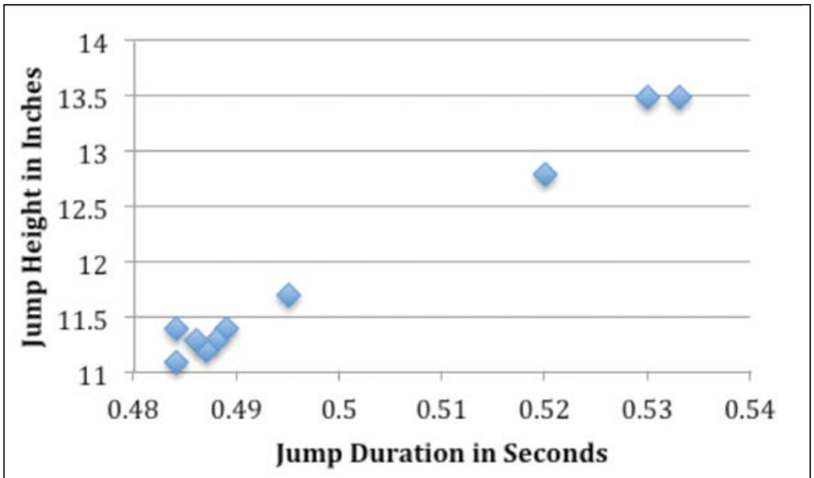

Fig. 3 The data from Brad's jumps (second experiment) appear not to be linear because of the behavior at the ends of the plot.

DEVON GUNTER, dgunter@ou.edu, is a PhD student in math education in the Jeannine Rainbolt College of Education at the University of Oklahoma, where he also serves as an adjunct lecturer in the math department. He is interested in inquiry-based learning and the secondary-tertiary transition.

more4U

For an additional scenario of students collecting jump data, go to **www.nctm.org/mt.**

The more4U content, an additional benefit, is for members only.